HW 9

① (a) Linear polarization:

$$\vec{E}(z,t) = \hat{\imath}\, E_{ox} \cos(kz - \omega t) + \hat{\jmath}\, E_{oy} \cos(kz - \omega t)$$

(this is general, but others work)

(b) Circular polarization:

$$\vec{E}(z,t) = \hat{\imath}\, E_{o} \cos(kz - \omega t) + \hat{\jmath}\, E_{o} \sin(kz - \omega t)$$

(c) Elliptical polarization:

$$\vec{E}(z,t) = \hat{\imath}\, E_{ox} \cos(kz - \omega t) + \hat{\jmath}\, E_{oy} \cos(kz - \omega t + \varepsilon)$$

(general)

② (a) $\frac{1}{2} I_o$

(b) The directions of polarization of the two polarizers are at 90° to each other. ("crossed")

(c) 45° to either polarizer

$$I = \tfrac{1}{2} I_o \cdot \cos^2(45°) \cdot \cos^2(45°) = \tfrac{1}{8} I_o$$

(d) $I = \frac{1}{2} I_o \cdot \left(\cos^2(9°)\right)^{10} \approx 0.39\, I_o$

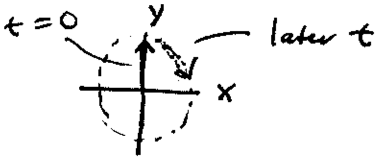
③ This happens at Brewster's angle:

$$\tan\theta_B = n_t/n_i = 1.33/1$$

$$\theta_B \approx 53.1^\circ$$

④ A half-wave plate retards the o-rays by $\frac{\lambda}{2}$ more than the e-rays. (The e-rays correspond to polarization along the vertical axis.)

Right circular enters:



$$\vec{E} = \hat{i}\, E_0 \cos(kz-\omega t) + \hat{j}\, E_0 \sin(kz-\omega t)$$

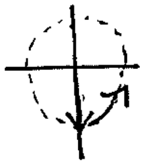
relative delay in E_{0x}:

$$\vec{E}_{final} = \hat{i}\, E_0 \cos(kz-\omega t+\pi) + \hat{j}\, E_0 \sin(kz-\omega t)$$

$$= -\hat{i}\, E_0 \cos(kz-\omega t) + \hat{j}\, E_0 \sin(kz-\omega t)$$

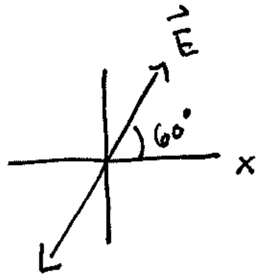
→ The $\vec{E}_{0x}$ is still shifted $\frac{\pi}{2}$ from $\vec{E}_{0y}$, but E_{0y} now lags instead of leads:

Left circularly Polarized



after $\lambda/2$ waveplate

⑤



(Begin with:)

$$\vec{E} = \hat{\imath}\,\frac{E_0}{2}\cos(kx-\omega t) + \hat{\jmath}\,\frac{\sqrt{3}E_0}{2}\cos(kx-\omega t)$$

The quarter-wave plate introduces a $\pi/2$ (or $\lambda/4$) shift of $\vec{E}_{0x}$ with respect to $\vec{E}_{0y}$.

↑ extraordinary (under $\vec{E}_{0x}$) — ↑ ordinary (under $\vec{E}_{0y}$)

$$\vec{E} = \hat{\imath}\,\frac{E_0}{2}\cos(kx-\omega t) + \hat{\jmath}\,\frac{\sqrt{3}}{2}E_0\cos\left(kx-\omega t + \frac{\pi}{2}\right)$$

$$\vec{E} = \hat{\imath}\,\frac{E_0}{2}\cos(kx-\omega t) - \hat{\jmath}\,\frac{\sqrt{3}}{2}E_0\sin(kx-\omega t)$$

↑ Left-handed elliptical polarization

Axis: $\varepsilon = \frac{\pi}{2} \Rightarrow \cos\varepsilon = 0 \Rightarrow \alpha = 0$

→ The axes are therefore on the the x- & y-axes, and $|E_{0y}| > |E_{0x}|$.

⇒ Major axis // y-axis

⑥ The polarizer will reduce the natural light intensity by 50%. The QWP will not reduce irradiance.

⑦ • Whenever you are looking through calcite but not through the polarizer, you see 2 images, one from light that is polarized ~~along the calcite's~~ perpendicular to the calcite's optic axis (o-rays) and the other (e-rays) having some component of their E-field // to the optic axis.

Looking through the polarizer:

- We are told that the polarizer only passes light with horizontal polarization.
- In (a) and (c), the double image is removed, because the light that goes through the filter (polarizer) has its E-field aligned with either the e-rays or o-rays.
  - → You can see that in (c), the image through the polarizer is not refracted from the text without calcite. Thus, (c) shows o-rays & (a) shows e-rays.
- In (b), the axis of the polarizer forms an angle with the crystal axis (angle of 45° to both o- & e-rays). Thus, we again see a double image of both o- & e-rays.